Transport Homework 8

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1 Problem Statement

1.1 8.8

At a particular axial station, velocity and temperature profiles for laminar flow in a parallel plate channel have the form:

$$u(y) = 0.75\left[1 - \frac{y}{y_o}^2\right] \quad (1)$$

$$T(y) = 5.0 + 95.66\left(\frac{y}{y_o}\right)^2 - 47.83\left(\frac{y}{y_o}\right)^4 \quad (2)$$

Determine corresponding values of the mean velocity, u_m, and mean (or bulk) temperature, T_m. Plot the velocity and temperature distributions. Do your values of u_m and T_m appear reasonable?

1.2 8.32

Batch processes are often used in chemical and pharmaceutical operations to achieve a desired chemical composition for the final product. Related heat transfer processes are typically transient, involving a liquid of fixed volume that may be heated from room temperature to a desired process temperature, or cooled from the process temperature to room temperature. Consider a batch process for which a pharmaceutical (the cold fluid, c) is poured into an insulated, highly agitated vessel (a stirred reactor) and heated by passing a hot fluid (h) through a submerged heat exchanger coil of thin-walled tubing and surface area As. The flow rate, $\dot{m}_h$, mean inlet temperature, $T_{h,i}$, and specific heat, $c_{p,h}$, of the hot fluid are known, as are the initial temperature, $T_{c,i}$ ¡ $T_{h,i}$, the volume, V_c, mass density ρ_c, c, and specific heat, $c_{v,c}$, of the pharmaceutical. Heat transfer from the hot fluid to the pharmaceutical is governed by an overall heat transfer coefficient U.
a) Starting from basic principles, derive expressions that can be used to determine the variation of T_c and $T_{h,o}$ with time during the heating process.
b) Consider a pharmaceutical of volume V_c = 1 m^3, density ρ_c = 1100 kg/m^3, specific heat $c_{v,c}$ = 2000 J/kgK, and an initial temperature of $T_{c,i}$ = 25C. A coiled tube of length L = 40 m, diameter D = 50 mm, and coil diameter C = 500 mm is submerged in the vessel, and hot fluid enters the tubing at $T_{h,i}$ = 200C and $\dot{m}_h$ = 2.4 kg/s. The convection coefficient at the outer surface of the tubing may be approximated as ho = 1000 W/m^{2}K, and the fluid properties are $c_{p,h}$ = 2500 J/kgK, h = 0.002 Ns/m2, k_h = 0.260 W/mK, and Pr_h = 20.

For the foregoing conditions, compute and plot the pharmaceutical temperature T_c and the outlet temperature $T_{h,o}$ as a function of time over the range $0 \leq t \leq 3600$s. How long does it take to reach a batch temperature of $T_c = 160$C? The process operator may control the heating time by varying $\dot{m}_h$. For $1 \leq \dot{m}_h \leq 5$ kg/s, explore the effect of the flow rate on the time t_c required to reach a value of $T_c = 160$C.

1.3 8.60

A hot fluid passes through a thin-walled tube of 10 mm diameter and 1 m length, and a coolant at $T_\infty = 25$C is in cross flow over the tube. When the flow rate is $\dot{m} = 18$ kg/h and the inlet temperature is $T_{m,i} = 85$C, the outlet temperature is $T_{m,o} = 78$C. Assuming fully developed flow and thermal conditions in the tube, determine the outlet temperature, $T_{m,o}$, if the flow rate is increased by a factor of 2. That is, $\dot{m} = 36$ kg/h, with all other conditions the same. The thermophysical properties of the hot fluid are $\rho = 1079$ kg/m^3, $c_p = 2637$ J/kgK, $\mu = 0.0034$ Ns/m^2, and k = 0.261 W/mK.

1.4 8.75

The temperature of flue gases flowing through the large stack of a boiler is measured by means of a thermocouple enclosed within a cylindrical tube as shown. The tube axis is oriented normal to the gas flow, and the thermocouple senses a temperature T_t corresponding to that of the tube surface. The gas flow rate and temperature are designated as m_g and T_g, respectively, and the gas flow may be assumed to be fully developed. The stack is fabricated from sheet metal that is at a uniform temperature T_s and is exposed to ambient air at T_∞ and large surroundings at T_{sur}. The convection coefficient associated with the outer surface of the duct is designated as h_o, while those associated with the inner surface of the duct and the tube surface are designated as h_i and h_t, respectively. The tube and duct surface emissivities are designated as ϵ_t and ϵ_s, respectively.
(a) Neglecting conduction losses along the thermocouple tube, develop an analysis that could be used to predict the error (T_g - T_t) in the temperature measurement.
(b) Assuming the flue gas to have the properties of atmospheric air, evaluate the error for T_t = 300C, $D_s = 0.6$ m, $D_t = 10$ mm, $m_g = 1$ kg/s, $T_\infty = T_{sur} = 27$C, $\epsilon_t = \epsilon_s = 0.8$, and $h_o =$ 25 W/m2K.

2 Problem Answer

2.1 8.8

$$\dot{m} = \rho \times A_c \times u_m = \rho \int_{A_c} u(y) dA_c \tag{3}$$

$$u_m = \frac{1}{A_c} \int_{A_c} u(y) dA_c, dA_c = dy, A_c = 2y_o \tag{4}$$

$$u_m = 0.50 m/s \tag{5}$$

$$\dot{E}_t = \dot{m} c_v T_m = \rho A_c u_m c_v T_m = \rho c_p \int_{A_c} (u(y) \times T(y)) dA_c \tag{6}$$

$$\text{Same process now: } T_m = 20.0C \tag{7}$$

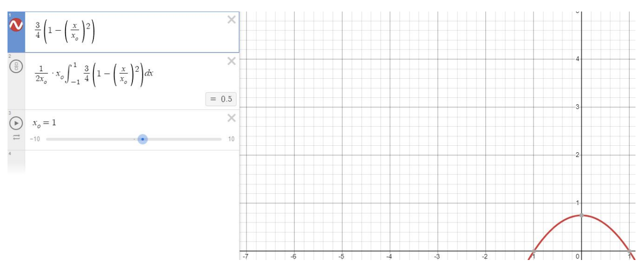

Figure 1: Velocity Profile

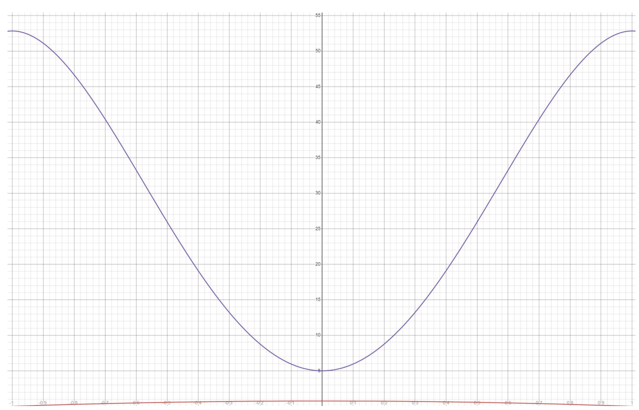

Figure 2: Temperature Profile

The velocity profile seems reasonable (equal areas above and below, roughly). The temperature profile is also reasonable.

2.2 8.32

a) Energy balance:

$$\frac{dU_c}{dt} = \rho_c V_c c_{v,c} \frac{dT_c}{dt} = q(t) \tag{8}$$

$$q(t) = \dot{m}_h c_{p,h}(T_{h,i} - T_{h,o}) = UA_s \Delta T_{l,m} \tag{9}$$

$$\Delta T_{l,m} = \frac{T_{h,i} - T_{h,o}}{ln\left[\frac{T_{h,i}-T_c}{T_{h,o}-T_c}\right]} \tag{10}$$

$$\dot{m}_h c_{p,h}(T_{h,i} - T_{h,o}) = UA_s \frac{T_{h,i} - T_{h,o}}{ln\left[\frac{T_{h,i}-T_c}{T_{h,o}-T_c}\right]} \tag{11}$$

$$\frac{1}{ln\left[\frac{T_{h,i}-T_c}{T_{h,o}-T_c}\right]} = \frac{\dot{m}_h c_{p,h}}{UA_s} \tag{12}$$

$$T_{h,o}(t) = T_c + (T_{h,i} - T_c)e^{\frac{-UA_s}{\dot{m}_h c_{p,h}}} \tag{13}$$

$$\tag{14}$$

This is expression for $T_{h,o}$.

$$\dot{m}_h c_{p,h}(T_{h,i} - T_c - (T_{h,i} - T_c)e^{\frac{-UA_s}{\dot{m}_h c_{p,h}}}) = \rho_c V_c c_{v,c} \frac{dT_c}{dt} \tag{15}$$

$$\frac{\dot{m}_h c_{p,h}}{\rho_c V_c c_{v,c}}(T_{h,i} - T_c - (T_{h,i} - T_c)e^{\frac{-UA_s}{\dot{m}_h c_{p,h}}}) = \frac{dT_c}{dt} \tag{16}$$

$$\frac{\dot{m}_h c_{p,h}(T_{h,i} - T_c)}{\rho_c V_c c_{v,c}}(1 - e^{\frac{-UA_s}{\dot{m}_h c_{p,h}}}) = \frac{dT_c}{dt} \tag{17}$$

$$\frac{\dot{m}_h c_{p,h}}{\rho_c V_c c_{v,c}}(1 - e^{\frac{-UA_s}{\dot{m}_h c_{p,h}}}) = \frac{dT_c}{(T_c - T_{h,i})} \tag{18}$$

$$\tag{19}$$

Integrating both sides and some mathematical rearrangement:

$$-ln\left(\frac{T_c - T_{h,i}}{T_{c,i} - T_{h,i}}\right) = \frac{\dot{m}_h c_{p,h}}{\rho_c V_c c_{v,c}}(1 - e^{\frac{-UA_s}{\dot{m}_h c_{p,h}}})t \tag{20}$$

This can be rearranged and exponentiated to isolate T_c. b) We have to first determine U, which is $(h_i^{-1} + h_o^{-1})^{-1}$, parallel resistor-like setup. The Reynold's number is $4\dot{m}/\pi\mu D$ which in this instance is 30600, meaning the flow is turbulent (since it is a cylindrical heat exchanger). Therefore the following relation is used:

$$h_i = \frac{k}{D}Nu_D; Nu_D = 0.023(Re_D^{4/5}Pr_h^{0.3}); h_i = 1140\text{W}/m^2K \tag{21}$$

Plugging that back into the formula to determine U yields 532 W/m^2K, which allows us to complete the $T_c(t)$ function from part a.

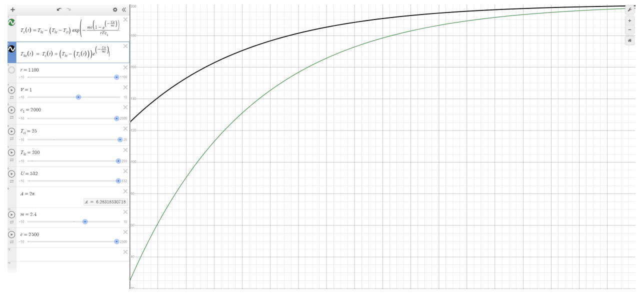

Figure 3: Temperature Profiles

Now we can easily find the temperature at which pharmaceuticals hit 160C (just by inputting x = 160 and finding the intersection point with T_c, which in this instance is 1266 seconds). We would expect that decreasing $\dot{m}_h$ will increase the time needed for the pharmaceuticals to get to 160C, and increasing the flow rate will have the opposite effect.

2.3 8.60

This can be evaluated using the following equation:

$$\frac{T_\infty - T_{m,o}}{T_\infty - T_{m,i}} = e^{\frac{-\pi D L \bar{U}}{\dot{m}_o c_p}} \tag{22}$$

All of these except $\bar{U}$ are given, thus it can be calculated as it is the only unknown - this works out to 52.1 W/m^2K. Once again, we can express U as the reciprocal of the sum of the reciprocals of inside and outside coefficients. Then we can find Reynold's number, which we have all the givens for - this turns out to be 186, which is laminar for internal flow. Due to laminar flow, doubling the flow rate will not change the internal coefficient, thus U is still 52.1. Now resolving for $T_{m,o}$ from the initial formula (we know all the other variables, so we can solve for it) we find that it is 81.4C, which is expected because pumping more hot fluid through the pipe would cause the other end to warm up (similar to the pharmaceutical example).

2.4 8.75

a) Assuming steady state (steady temperature) we can say $q_{rad} = q_{conv}$. Putting that in a more sophisticated form:

$$h_t A_t (T_g - T_t) = \epsilon_t \sigma A_t (T_t^4 - T_s^4); T_g = T_t + \frac{\epsilon_t \sigma}{h_t}(T_t^4 - T_s^4) \tag{23}$$

We need to determine T_s which requires a wall balance: $q_{conv,i} = q_{conv,o} + q_{rad}$. This complicates the formula for T_g a little:

$$T_g = T_s + \frac{h_i}{h_o}(T_s - T_\infty) + \frac{\epsilon_s \sigma}{h_i}(T_t^4 - T_s^4) \tag{24}$$

Now, both T_g and T_s can be solved simultaneously.
b) Using A-4, T_g at around 600K, 1 atm of pressure data values for air. As follows:

$$h_t = \bar{N}u_D \times \frac{k}{D_t} \tag{25}$$

$$\bar{N}u_D = 0.26 Re_{Dt}^{0.6} Pr^{0.37} \text{ via Table 7.4, as shown next} \tag{26}$$

$$Re_{Dt} = \frac{4\dot{m}_g D_t}{\pi \mu D_s^2} = 1157 \tag{27}$$

$$h_t = 73 W/m^2 K \tag{28}$$

$$T_g = T_t + \frac{\epsilon_t \sigma}{h_t}(T_t^4 - T_s^4) \tag{29}$$

$$T_g = 640 - 6.214 \times 10^{-10} T_s^4 \tag{30}$$

$$Re_{Ds} = \frac{4\dot{m}_g}{\pi \mu D_s} = 6.94 \times 10^4 \tag{31}$$

$$\bar{N}u_{D1} = 0.023 Re_{Ds}^{0.8} Pr^{0.3} \tag{32}$$

$$h_i = \bar{N}u_{D1} \times \frac{k}{D_s} = 12 W/m^2 K \tag{33}$$

$$T_g = T_s + \frac{h_i}{h_o}(T_s - T_\infty) + \frac{\epsilon_s \sigma}{h_i}(T_t^4 - T_s^4) \tag{34}$$

$$T_g = -655.6K + 3.083 T_s + 3.78 \times 10^{-9} T_s^4 \tag{35}$$

$$\tag{36}$$

Iterative approach until results from both equations are equal: $T_s = 388$K, $T_g = 626$K while T_t given to be 300C or 573K, so 53 degree difference!