Transport Homework 7

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November 7 2022

1 Problem Statement

1.1 6.82

A 2-mm-thick layer of water on an electrically heated plate is maintained at a temperature of $T_w = 340$ K, as dry air at $T_\infty = 300$ K flows over the surface of the water (case A). The arrangement is in large surroundings that are also at 300 K.
a) If the evaporative flux from the surface of the water to the air is $n_A^" = 0.030 \text{kg/m}^2\text{s}$, what is the corresponding value of the convection mass transfer coefficient? How long will it take for the water to completely evaporate?
b) What is the corresponding value of the convection heat transfer coefficient and the rate at which electrical power must be supplied per unit area of the plate to maintain the prescribed temperature of the water? The emissivity of water is $\epsilon_w = 0.95$.
c) If the electrical power determined in part (b) is maintained after complete evaporation of the water (case B), what is the resulting temperature of the plate, whose emissivity is $\epsilon_P = 0.60$?

1.2 7.20

The roof of a refrigerated truck compartment is of composite construction, consisting of a layer of foamed urethane insulation ($t_2 = 50$ mm, $k_i = 0.026$ W/mK) sandwiched between aluminum alloy panels ($t_1 = 5$ mm, $k_p = 180$ W/mK). The length and width of the roof are L = 10 m and W = 3.5 m, respectively, and the temperature of the inner surface is $T_{s,i} =$ -10C. Consider conditions for which the truck is moving at a speed of V = 105 km/h, the air temperature is $T_\infty = 32$C, and the solar irradiation is $G_S = 750$ W/m^2. Turbulent flow may be assumed over the entire length of the roof.
a) For equivalent values of the solar absorptivity and the emissivity of the outer surface ($\alpha_S = \epsilon = 0.5$), estimate the average temperature $T_{s,o}$ of the outer surface. What is the corresponding heat load imposed on the refrigeration system?
b) A special finish ($\alpha_S = 0.15$; $\epsilon = 0.8$) may be applied to the outer surface. What effect would such an application have on the surface temperature and the heat load?
c) If (with $\alpha_S = \epsilon = 0.5$) the roof is not insulated ($t_2 = 0$), what are the corresponding values of the surface temperature and the heat load?

1.3 7.24

Steel (AISI 1010) plates of thickness $\delta = 6$ mm and length L = 1 m on a side are conveyed from a heat treatment process and are concurrently cooled by atmospheric air of velocity u_∞ = 10 m/s and T_∞ = 20C in parallel flow over the plates. For an initial plate temperature of T_i = 300C, what is the rate of heat transfer from the plate? What is the corresponding rate of change of the plate temperature? The velocity of the air is much larger than that of the plate.

2 Problem Solutions

2.1 6.82

a) Knowing the temperature of the water we can find the density of the water and the density of the saturated vapor, as well as h_{fg}, density of air, c_P of air, k of air, and D_{AB} of air. The values are listed below (water then air):

$$\rho_f = 979 \text{ kg/m}^3; \rho_{A,sat} = 0.174 \text{ kg/m}^3; h_{fg} = 2342 \text{ kJ/kg} \tag{1}$$

$$\rho = 1.08 \text{ kg/m}^3; c_P = 1008 \text{ J/kg}; k = 0.028 \text{ W/mK}; D_{AB} = 0.29 \times 10^{-4} \text{ m}^2\text{/s} \tag{2}$$

We can use the following expression to evaluate part a:

$$n''_A = h_m(\rho_{A,sat} - \rho_{A,\infty}) \tag{3}$$

$$0.030 \text{ kg/sm}^2 = h_m(0.174 \text{ kg/m}^3) \tag{4}$$

$$h_m = 0.172 \text{ m/s} \tag{5}$$

$$\rho_f \int_{\delta_i}^{0} d\delta = -n''_A \int_0^t dt \tag{6}$$

$$t = \frac{\rho_f \delta_i}{n''_A} = \frac{979 \text{ kg/m}^3 \times 0.002\text{m}}{0.03 \text{ kg/sm}^2} \tag{7}$$

$$t = 65.3 \text{ s} \tag{8}$$

b) Setup to solve part b relies on the two following equations:

$$h = \frac{k h_m}{D_{AB} Le^n}, n = 1/3 \tag{9}$$

$$Le = \frac{k}{\rho c_P D_{AB}} \tag{10}$$

$$h = 173 \text{ W/Km}^2 \tag{11}$$

Solving for the power used will be the sum of the various q" values.

$$P''_{elec} = q''_{evap} + q''_{conv} + q''_{rad} = n''_A h_{fg} + h(T_w - T_\infty) + \epsilon_w \sigma (T_w^4 - T_{sur}^4) \tag{12}$$

$$P''_{elec} = 77,464 \text{ W/m}^2 \tag{13}$$

c) Here we can use the power determined in part b and solve for the temperature of the plate by only considering the convection and radiation terms (as there is no more evaporation).

$$P^{"}_{elec} = h(T_p - T_\infty) + \epsilon_p \sigma (T_p^4 - T_{sur}^4) \tag{14}$$

Using fsolve, T_p was calculated to be 702K.

2.2 7.20

a) Assuming the temperature of the air close to the outer surface of the truck is around 300 K, the following properties apply:

$$c_P = 1007 \text{ J/kgK}; \nu = 15.89 \times 10^6 \text{ m}^2/\text{s}; P_r = 0.707; k = 0.0263 \text{W/mK} \tag{15}$$

Assuming the flow of air is turbulent the following formula applies:

$$q^{"}_{conv} = \bar{h}(T_\infty - T_{s,o}); \bar{h} = \frac{k}{L} 0.037 {Re_L}^{4/5} {P_r}^{1/3}; Re_L = \frac{u_\infty L}{\nu} \tag{16}$$

The truck is at steady state so the following formula applies:

$$q^{"}_{cond} = \alpha_S G_S + q^{"}_{conv} - E = \frac{T_{s,o} - T_{s,i}}{R_{tot}} = \frac{T_{s,o} - T_{s,i}}{2R^{"}_p + R^{"}_i} \tag{17}$$

$$R^{"}_p = \frac{t_1}{k_p}; R^{"}_i = \frac{t_2}{k_i} \tag{18}$$

$$q^{"}_{cond} = \alpha_S G_S + \frac{k}{L} 0.037 \left(\frac{u_\infty L}{\nu} \right)^{4/5} {P_r}^{1/3} (T_\infty - T_{s,o}) - \epsilon \sigma {T_{s,o}}^4 = \frac{T_{s,o} - T_{s,i}}{2\frac{t_1}{k_p} + \frac{t_2}{k_i}} \tag{19}$$

All these values except $T_{s,o}$ are known, so we can use fsolve in MATLAB to compute this. This yields (with initial guess of 300K) 306.8K or 33.8C. Heat loss is $q^{"}_{cond} \times$ L $\times$ W which would be 797W.
b) Solving for $T_{s,o}$ with the altered values of α_S and ϵ using fsolve. This yields 300.1K (23.1C) and a heat loss of 675W.
c) Solving for $T_{s,o}$ with no insulation resistance term using fsolve. This yields 263.1K or -9.9C and a heat loss of 90680W.

2.3 7.24

Via Table A-1, the following properties apply for the steel at 573K (via interpolation between 400K and 600K):

$$k_p = 49.2 \text{ W/mK}; c = 549 \text{ J/kgK}, \rho = 7832 \text{ kg/m}^3 \tag{20}$$

Via Table A-4, the following properties apply for the air (assuming air close to the material is around 450K):

$$\nu = 32.4 \times 10^{-6}\ \mathrm{m^2/s}, k = 0.0373\ \mathrm{W/mK}, P_r = 0.686. \tag{21}$$

We can use this ν value to calculate Re_L, which is $\frac{u_\infty L}{\nu}$. This works out to be 308,641 which is less than 5×10^5, so flow is laminar. Thus we use the following formula:

$$\bar{N}u_L = 0.664 {Re_L}^{1/2} {P_r}^{1/3} = 325.3 \tag{22}$$

Now we can determine $\bar{h}$ via the following formula:

$$\bar{h} = \frac{k}{L} \bar{N}u_L = 12.1\ \mathrm{W/Km^2} \tag{23}$$

We know that q can be expressed as follows:

$$q = 2\bar{h}A_s(T_i - T_\infty) = 6780\ \mathrm{W} \tag{24}$$

Now we can perform an energy balance as follows:

$$\rho\delta L^2 c \frac{dT}{dt}|_i = -q = 2\bar{h}L^2(T_i - T_\infty) \tag{25}$$

$$\frac{dT}{dt}|_i = \frac{-q}{\rho\delta c}, \text{ since L = 1 m it can effectively be ignored} \tag{26}$$

$$\frac{dT}{dt}|_i = \frac{-6780\ \mathrm{J/s}}{7832\ \mathrm{kg/m^3} \times 0.006\ \mathrm{m} \times 549\ \mathrm{J/kgK}} \tag{27}$$

$$\frac{dT}{dt}|_i = -0.263\ \mathrm{K/s} \tag{28}$$