Transport Homework 6

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1 Problem Statement

1.1 14.34

Consider a spherical organism of radius ro within which respiration occurs at a uniform volumetric rate of Na = -k0. That is, oxygen (species A) consumption is governed by a zero-order, homogeneous chemical reaction.
(a) If a molar concentration of CA(ro) = CA,o is maintained at the surface of the organism, obtain an expression for the radial distribution of oxygen, CA(r), within the organism. From your solution, can you discern any limits on applicability of the result?
(b) Obtain an expression for the rate of oxygen consumption within the organism.
(c) Consider an organism of radius ro = 0.10 mm and a diffusion coefficient for oxygen transfer of DAB = 10^{-8} m^2/s. If CA,o = 5 × 10^{-5} kmol/m^3 and k0 = 1.2×10^{-4} kmol/sm^3, what is the molar concentration of O2 at the center of the organism?

1.2 14.38

As an employee of the Los Angeles Air Quality Commission, you have been asked to develop a model for computing the distribution of NO2 in the atmosphere. The molar flux of NO2 at ground level, $N^{"}_{A.o}$, is presumed known. This flux is attributed to automobile and smoke stack emissions. It is also known that the concentration of NO2 at a distance well above ground level is zero and that NO2 reacts chemically in the atmosphere. In particular, NO2 reacts with unburned hydrocarbons (in a process that is activated by sunlight) to produce PAN (peroxyacetylnitrate), the final product of photochemical smog. The reaction is first order, and the local rate at which it occurs may be expressed as Na = -k1CA.
(a) Assuming steady-state conditions and a stagnant atmosphere, obtain an expression for the vertical distribution CA(x) of the molar concentration of NO2 in the atmosphere.
(b) If an NO2 partial pressure of pA = 2×10^{-6} bar is sufficient to cause pulmonary damage, what is the value of the ground level molar flux for which you would issue a smog alert? You may assume an isothermal atmosphere at T = 300 K, a reaction coefficient of k1 = 0.03 s^{-1}, and an NO2–air diffusion coefficient of DAB = 0.15 × $10^{-4}$$m^2$/s.

1.3 6.18

Consider airflow over a flat plate of length L = 1 m under conditions for which transition occurs at xc = 0.5 m based on the critical Reynolds number, Rex,c = 5 × 10^5.

(a) Evaluating the thermophysical properties of air at 350 K, determine the air velocity.
(b) In the laminar and turbulent regions, the local convection coefficients are, respectively,

$$h_{lam}(x) = C_{lam}x^{-0.5} \text{ and } h_{turb} = C_{turb}x^{-0.2} \tag{1}$$

where, at T = 350 K, Clam = 8.845 W/$m^{3/2}$ K, Cturb = 49.75 W/$m^{1.8}$ K, and x has units of m. Develop an expression for the average convection coefficient, lam(x), as a function of distance from the leading edge, x, for the laminar region, 0 ≤ x ≤ xc.
(c) Develop an expression for the average convection coefficient, turb(x), as a function of distance from the leading edge, x, for the turbulent region, xc ≤ x ≤ L.
(d) On the same coordinates, plot the local and average convection coefficients, hx and , respectively, as a function of x for 0 ≤ x ≤ L.

1.4 6.39

Forced air at T = 25C and V = 10 m/s is used to cool electronic elements on a circuit board. One such element is a chip, 4 mm by 4 mm, located 120 mm from the leading edge of the board. Experiments have revealed that flow over the board is disturbed by the elements and that convection heat transfer is correlated by an expression of the form

$$Nu_x = 0.04Re_x^{0.85}Pr^{1/3} \tag{2}$$

Estimate the surface temperature of the chip if it is dissipating 30 mW.

2 Problem Solutions

2.1 14.34

a) We can express the system (and evaluate for C_A in the problem as follows:

$$\frac{D_{AB}}{r^2}\frac{d}{dr}\left(r^2\frac{dC_A}{dr}\right) - k_0 = 0 \tag{3}$$

$$r^2\frac{dC_A}{dr} = \frac{k_0 \times r^3}{3D_{AB}} + C_1 \tag{4}$$

$$C_A = \frac{k_0 \times r^2}{6D_{AB}} - \frac{C_1}{r} + C_2 \tag{5}$$

C_1 must be 0 because at r = 0, C_A is given to be finite. Knowing that $C_A(r_o) = C_{A,o}$ allows us to determine C_2, which is the following:

$$C_2 = C_{A,o} - \frac{k_0 \times r^2}{6D_{AB}} \tag{6}$$

Thus C_A is the following:

$$C_A = C_{A,o} - \frac{k_0}{6D_{AB}} \times (r_o^2 - r^2) \tag{7}$$

C_A cannot be less than zero anywhere, which means $C_{A,o} \geq \frac{k_0 \times r_o^2}{6D_{AB}}$.
b) Oxygen consumption occurs at a volumetric rate of k_0, which means total respiration rate is just k_0 multiplied by volume. So it is $\frac{4}{3}\pi r_o^3 k_0$. c) Here we can plug all these numbers into equation 7 and substitute r for 0. This yields a concentration of $3 \times 10^{-5} \frac{kmol}{m^3}$.

2.2 14.38

a) We need to express the vertical distribution C_A(x) of NO2. We know that infinitely far away, the concentration of NO2 is 0, and the molar flux at ground is known. This can be expressed as follows:

$$\left(\frac{dC_A}{dx}\right)_{x=0} = \frac{-N^{"}_{A,o}}{D_{AB}} \tag{8}$$

Since at $x = \infty$ C_A is 0 (where the general equation is $C_A(x) = C_1 e^{mx} + C_2 e^{-mx}$, C_1 is 0. C_2 is equal to $\frac{N^{"}_{A,o}}{m \times D_{AB}}$, where m is $(k_1/D_{AB})^{1/2}$. So C_A(x) is as follows:

$$C_A(x) = \frac{N^{"}_{A,o}}{m \times D_{AB}} e^{-mx} \tag{9}$$

b) We know the concentration $C_A(0)$. Using the ideal gas law we can write the following equivalency:

$$p_A(0) = C_A(0) \times R \times T = \frac{N^{"}_{A,o}}{m \times D_{AB}} \times R \times T \tag{10}$$

Rearranging to solve for $N^{"}_{A,o}$ leads to this:

$$N^{"}_{A,o} = \frac{m \times D_{AB} \times p_A(0)}{R \times T} \tag{11}$$

Now this can be solved numerically, which yields $N^{"}_{A,o} = 5.38 \times 10^{-11} \frac{kmol}{sm^2}$.

2.3 6.18

Via Table A.4 at T = 350K, k = 0.030 W/mK, nu = 20.92 $\times 10^{-6} m^2/s, P_r = 0.700$.
a) $\text{Re}_{x,c} = \frac{u_\infty x_c}{\nu} = 5 \times 10^5$. Backsolving for u_∞ using the ν value identified from the table yields a velocity of 20.9 m/s.
b) $\bar{h}_{lam}(x) = \frac{1}{x}\int_0^x h_{lam}(x)dx$. We also know $h_{lam}(x) = C_{lam}x^{-0.5}$. Via integration we can say $\bar{h}_{lam}(x) = \frac{2}{x}C_{lam}x^{0.5} = 2C_{lam}x^{-0.5} = 2h_{lam}(x)$.
c) $\bar{h}_{turb}(x) = \frac{1}{x}\left(\int_{x_c}^x h_{turb}(x)dx + \int_0^{x_c} h_{lam}(x)dx\right)$. We can evaluate this in the same way we evaluated part b. The laminar integral works out to be $2C_{lam}x_c^{0.5}$. The turbulent integral works out to $1.25C_{turb} \times (x^{0.8} - x_c^{0.8})$. So $\bar{h}_{turb}(x) = \frac{1}{x}\left[2C_{lam}x_c^{0.5} + 1.25C_{turb} \times (x^{0.8} - x_c^{0.8})\right]$.
d) Plot and code attached in zip file.

2.4 6.39

We use Table A.4 here again. Tf = 308K (assuming Ts is 45C), k = 0.027 W/mK, nu = 16.69 $\times 10^{-6} m^2/s, P_r = 0.703$. Assuming steady state, the q of convection must be equal to energy generation, so 30W of q convection. Using Newton's law of cooling ($T_s = T_\infty + q_{conv}/(\bar{h}A)$) and then evaluating. We are assuming that average transfer coefficient is the same as the local one. First we need to find the Nusselt number $Nu_x = \frac{h_x \times x}{k} = A[\frac{V}{\nu}]^{0.85} \times P_r^{1/3}$. We can then isolate and solve for h_x like so: $h_x = A\frac{k}{L}[\frac{VL}{\nu}]^{0.85} \times P_r^{1/3}$. Plugging in all the values to solve for h_x yields 107W/m^{2}K. Now solving for surface temperature:

$$T_s = T_\infty + \frac{\text{Heat Dissipated}}{h_x \times A} \quad (12)$$

With the values found/known in this problem, T$_s$ = 42.5C.