Transport Homework 5

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1 Problem Statement

1.1 14.11

An open pan of diameter 0.2 m and height 80 mm (above water at 27C) is exposed to ambient air at 27C and 25 percent relative humidity. Determine the evaporation rate, assuming that only mass diffusion occurs. Determine the evaporation rate, considering bulk motion.

1.2 14.15

A thin plastic membrane is used to separate helium from a gas stream. Under steady-state conditions the concentration of helium in the membrane is known to be 0.02 and 0.005 kmol/m^3 at the inner and outer surfaces, respectively. If the membrane is 1 mm thick and the binary diffusion coefficient of helium with respect to the plastic is $10^{-9}m^2/s$, what is the diffusive flux?

1.3 14.28

Ultra-pure hydrogen is required in applications ranging from the manufacturing of semiconductors to powering fuel cells. The crystalline structure of palladium allows only the transfer of atomic hydrogen (H) through its thickness, and therefore palladium membranes are used to filter hydrogen from contaminated streams containing mixtures of hydrogen and other gases. Hydrogen molecules (H_2) are first adsorbed onto the palladium's surface and are then dissociated into atoms (H), which subsequently diffuse through the metal. The H atoms recombine on the opposite side of the membrane, forming pure H_2. The surface concentration of H takes the form C$_H$ = $K_s p_{H^{0.5}{}_2}$, where Ks = 1.4 kmol/$m^3\times$ bar$^{0.5}$ is known as Sievert's constant. Consider an industrial hydrogen purifier consisting of an array of palladium tubes with one tube end connected to a collector plenum and the other end closed. The tube bank is inserted into a shell. Impure H_2 at T = 600 K, p = 15 bars, xH_2 = 0.85 is introduced into the shell while pure H_2 at p = 6 bars, T = 600 K is extracted through the tubes. Determine the production rate of pure hydrogen (kg/h) for N = 100 tubes which are of inside diameter D = 1.6 mm, wall thickness t = 75 m, and length L = 80 mm. The mass diffusivity of hydrogen (H) in palladium at 600 K is approximately D$_A B = 7\times10^{-9}m^2/s$.

1.4 14.32

A platinum catalytic reactor in an automobile is used to convert carbon monoxide to carbon dioxide in an oxidation reaction of the form 2CO + $O_2 \longrightarrow$ 2CO_2. Species transfer between the catalytic surface and the exhaust gases may be assumed to occur by diffusion in a film of thickness L = 10 mm. Consider an exhaust gas that has a pressure of 1.2 bars, a temperature of 500C, and a CO mole fraction of 0.0012. If the reaction rate constant of the catalyst is $k_1^{"}$ = 0.005 m/s and the diffusion coefficient of CO in the mixture is 10^{-4} m^2/s, what is the molar concentration of CO at the catalytic surface? What is the rate of removal of CO per unit area of the catalyst? What is the removal rate if $k_1^{"}$ is adjusted to render the process diffusion limited?

2 Problem Solutions

2.1 14.11

Using diffusion only, the following expression can be used:

$$N_{A,x} = N_{A,x}^{"} \times A = -D_{AB} \times A \times \frac{dC_A}{dx} \tag{1}$$

We know relative humidity and can thus reformat the expression into the following:

$$N_{A,x} = N_{A,x}^{"} \times A = -D_{AB} \times A \times \frac{C_{A,\infty} - C_{A,s}}{L} = \frac{D_{AB} \times A}{L} \times C_{A,sat}(1 - \text{rel. humidity}) \tag{2}$$

Plugging the numbers in yields a value of $N_{A,x}$ = 1.087 × 10⁻8 kmol/s (D_{AB} and specific volume [to calculate $C_{A,sat}$] found in Table A6.) Accounting for bulk motion (convective effects) changes the original formula to this:

$$N_{A,x} = N_{A,x}^{"} \times A = \frac{D_{AB} \times C \times A}{L} \times \ln \frac{1 - x_{A,L}}{1 - x_{A,0}} \tag{3}$$

C can be found via ideal gas law (here it is 0.04063 kmol/m^3). Then $x_{A,0}$ can be found through saturated pressure over total pressure. $x_{A,L}$ is found similarly just by multiplying $x_{A,0}$ by relative humidity. Saturated pressure for water was found in Table A8. Performing the computations yields 1.107 × 10⁻8 kmol/s, slightly higher than without accounting for bulk motion which is expected.

2.2 14.15

$$N_{A,x}^{"} = D_{AB} \times \frac{C_{A,1} - C_{A,2}}{L} \tag{4}$$

Directly plugging the numbers in yields $N_{A,x}^{"}$ = 1.5 × 10⁻8 kmol/sm^2

2.3 14.28

D_{AB} of hydrogen in palladium is $7 \times 10^{-9} m^2/s$. We can then use Sievert's constant to find the concentration of hydrogen gas on the outer and inner tube surface.

$$C_H = 1.4 \frac{\text{kmol}}{\text{m}^3\text{bar}^{1/2}} \times (xH_2 \times \text{p})^{1/2} \tag{5}$$

This yields a $C_H, 0$ of 5 kmol/m^3 and a C_H, i of 3.43 kmol/m^3. Then, the following two equations are used as diffusion resistances (cylindrical coordinates) for the wall and tube-end respectively.

$$R_{wall} = \frac{\ln(r_1/r_2)}{2\pi L D_{AB}} \tag{6}$$

$$R_{end} = \frac{\text{thickness}}{D_{AB} A_c} \tag{7}$$

Thus R_{wall} is 25.5 $\times 10^6 s/m^3$ and R_{end} is 5.33 $\times 10^9 s/m^3$. N_H can now be expressed like so:

$$N_H = \frac{C_{H,0} - C_{H,i}}{R_{wall}} + \frac{C_{H,0} - C_{H,i}}{R_{end}} = 61.9 \times 10^{-9} \frac{\text{kmol}}{s} \tag{8}$$

Now we can solve for N_{H_2} in terms of kmol per second and then convert to kilograms per hour using the 100 tubes, 3600 seconds per hour, and 2 kilograms per kmol.

$$N_H = 2N_{H_2} \longrightarrow N_{H_2} = 30.95 \times 10^{-9} \frac{\text{kmol}}{s} \tag{9}$$

$$30.95 \times 10^{-9} \frac{\text{kmol}}{s} \times 2 \frac{\text{kg}}{\text{kmol}} \times 3600 \frac{s}{h} \times 100 \text{ tubes} = 0.022 \frac{kg}{h} \tag{10}$$

2.4 14.32

$$x_A(0) = \frac{x_{A,L}}{1 + (Lk_1^{"}/D_{AB})} = 0.0008 \tag{11}$$

$$N_{A,s}^{"} = -N_A^{"}(0) = \frac{k_1^{"} \times C \times x_{A,L}}{1 + (Lk_1^{"}/D_{AB})} \tag{12}$$

Using C = p/RT with the values we're given, we can solve 12, which yields 7.48 $\times 10^{-8}$ kmol/sm^2. If the process is diffusion limited, $Lk_1^{"}/D_{AB}$ is much greater than one, so we can ignore the +1 in the denominator we find in formula 12. Also, the $k_1^{"}$s will cancel out as a result of this simplification. This yields the following:

$$N_{A,s}^{"} = \frac{C \times x_{A,L}}{(L/D_{AB})} = 2.24 \times 10^{-7} \frac{\text{kmol}}{\text{sm}^2} \tag{13}$$