Transport Homework 3

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October 17 2022

1 Problem Statement

1.1 Problem 5.12

Thermal energy storage systems commonly involve a packed bed of solid spheres, through which a hot gas flows if the system is being charged, or a cold gas if it is being discharged. In a charging process, heat transfer from the hot gas increases thermal energy stored within the colder spheres; during discharge, the stored energy decreases as heat is transferred from the warmer spheres to the cooler gas. Consider a packed bed of 75-mm-diameter aluminum spheres (2700 kg/m^3, c = 950 J/kg K, k = 240 W/m K) and a charging process for which gas enters the storage unit at a temperature of Tg,i = 300 C. If the initial temperature of the spheres is Ti = 25 C and the convection coefficient is h = 75 W/m^2 K, how long does it take a sphere near the inlet of the system to accumulate 90 percent of the maximum possible thermal energy? What is the corresponding temperature at the center of the sphere? Is there any advantage to using copper instead of aluminum?

1.2 Question 2

Consider a spherical reactor similar to the one in homework 3. The reactor has a radius R_1=1 m and the thickness of the reactor wall is negligible. At time 0, the temperature of the reaction media is set at 300 K. At that point, an endothermic reaction starts to take place inside the reactor and consumes $\dot{q} = -10^4$ W/m^3 of heat uniformly through the reactor. The thermal conductivity, k, of the reaction media (reactants and products) is 100 W/(K×m). The reactor wall is kept at a fixed temperature of 300 K throughout the reaction. The radial temperature profile across the reactor can be calculated using the heat diffusion equation:

$$k\left(\frac{1}{r^2}\frac{\partial}{\partial r}r^2\frac{\partial T}{\partial R}\right) + \dot{q} = \rho c\frac{\partial T}{\partial t} \quad (1)$$

$\dot{q}$ = volumetric heat consumption, $\rho = 1000\ \frac{kg}{m^3}$, c = 4000 $\frac{J}{kgK}$.
Use a finite difference approach to write a set of N ordinary differential equations (ODEs) that can be used to solve for the radial temperature profile T(r,t) at N r_j points along the reactor radius as a function of time. Make sure to identify the equations used for the internal points in both reactor regions (i.e. the reaction media and the reactor walls), as well as for the boundary conditions. Use Matlab to solve your system of ODEs and generate a surface plot of temperature vs. r position vs. time for the first 10,000 s.

2 Problem Solutions

2.1 5.12

We can use the lumped capacitance approximation for this problem. Biot number is less than 0.1 (radius of 37.5 mm, so Lc is 12.5 mm = 0.0125 m, k is 240 and h is 75 yields Biot number of 3.9 * 10^-3.

$$\frac{Q}{Q_{max}} = 1 - exp\left(\frac{-t}{\tau_t}\right) \tag{2}$$

Solving for $\tau_t = \frac{Dc}{6h}$ yields 427.5 seconds. Now we need to find the time it takes for $1 - exp\left(\frac{-t}{\tau_t}\right)$ to equal 0.90 due to the 90 percent of maximum possible thermal energy accumulation. Simplifying the expression by subtracting 1 from both sides and taking the natural log of both sides allows us to plug in the τ value and determine that t = 984 seconds.
Solving for the temperature at the center of the sphere, we can use the following formula:

$$\frac{T - T_\infty}{T_i - T_\infty} = exp\left(\frac{6ht}{\rho Dc}\right) \tag{3}$$

Rearranging and solving for T yields this equation:

$$T = T_\infty + (T_i - T_\infty)exp\left(\frac{6ht}{\rho Dc}\right) = 272.5°C \tag{4}$$

To compare copper and aluminum with regards to heat storing capabilities, we need to compare the product of their respective densities and specific heats. For copper ρ is 8960 kg/m^3, c is 376.8 J/kg K. For aluminum respectively these are 2700 kg/m^3 and 921 J/kg K. Dividing the product of copper's density and specific heat capacity by those of aluminum yields 1.358, or around a 35.8 percent increase in thermal energy storage.

2.2 Problem 2

$$\alpha\left(\frac{\partial^2 T}{\partial r^2} + \frac{2}{r}\frac{\partial T}{\partial r}\right) + \frac{\dot{q}}{\rho c} = \frac{\partial T}{\partial t} \tag{5}$$

$$\alpha\left[\left(1 + \frac{1}{j}\right)T_{j+1} - 2T_j + \left(1 - \frac{1}{j}\right)T_{j-1}\right] + \frac{\dot{q}}{\rho c} = \frac{\partial T}{\partial t} \tag{6}$$

$$\tag{7}$$

Boundary Conditions: the initial temperature everywhere is 300K, the temperature at the center (j=1 vs j=2) is approximately the same, the temperature at the surface j = N+1 is always 300K.

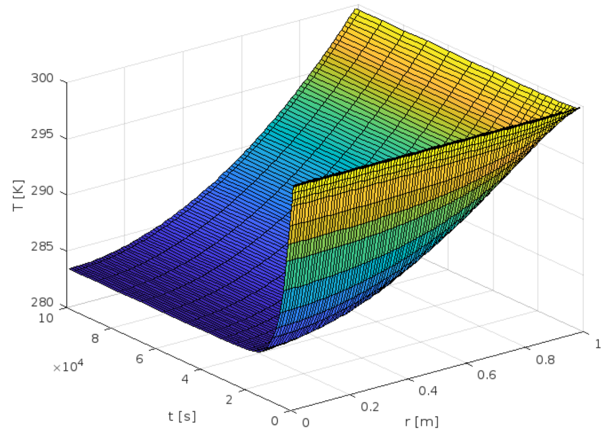

Figure 1: Time vs Radius vs Temperature