Transport Homework 2

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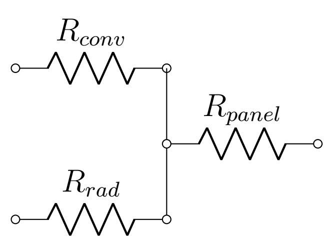
1 Problem Statement

Consider a house with a 100 m^2 roof covered with solar panels of 10 mm thickness. Assume that the house receives a solar irradiation of G = 1000 W/m^2 out of which only half is absorbed by the solar panels. 20 percent of the absorbed irradiation is converted into electricity and the rest is transferred in the form of heat to the top surface of the roof. The absorptivity and emissivity of the panel is 0.5. The house is kept at a constant temperature of 20 °C and the outside environment is at 40 °C. The wind provides convective heat exchange between the roof and the environment. The heat conductivity of the roof is 1.5 W/(mK).

2 Problem 1 Parts and Solutions

2.1 Part 1

Estimate the convective heat transfer coefficient, h, between the solar panels and the external environment given that the top surface of the panel was measured to be 28 °C. Assume that the bottom surface is kept at the same temperature of the room (T_{in} = 20 °C). Top surface in Kelvin: 301K. Surrounding temperature in Kelvin: 313K.



$$k\frac{T_i - T_s}{L} + G_{rad} = h_{rad}[T_s - T_{sur}] + h_{conv}(T_s - T_\infty) \tag{1}$$

$$q''_{rad} = \epsilon\sigma[T_s^4 - T_{sur}^4]; q''_{conv} = h_{conv}(T_s - T_\infty) \tag{2}$$

$$R_{conv} = \frac{1}{h_{conv}A}; R_{rad} = \frac{1}{h_{rad}A}; R_{panel} = \frac{L_{panel}}{k_{panel}A} \tag{3}$$

$$R_{tot} = \left[\frac{1}{h_{conv}A} + \frac{1}{h_{rad}A}\right]^{-1} + \frac{L_{panel}}{k_{panel}A} \tag{4}$$

The heat will flow towards the room (hot to cold). At the surface the temperature is 28 °C, and we can say that the energy balance at the surface is $q''_{G_{rad}} + q''_{cond} - q''_{conv} - q''_{rad} = 0$, where the G-radiation term is positive because the sun is heating up the panel while convection and conduction are cooling it. G_{rad} = 400W/m^2 (1000W from sun, 500 absorbed, 400 left as heat). Plugging in values into the first part of equation (1) yields -800W/m^2. h$_{rad}$ = $\epsilon\sigma(T_s + T_{sur})(T_s^2 + T_{sur}^2) = 0.004596$W/$m^2$. $T_s - T_\infty$ = -12°C. Setting up to solve for h_{conv}:

$$\frac{k\frac{T_i - T_s}{L} + G_{rad} - h_{rad}[T_s - T_{sur}]}{(T_s - T_\infty)} = h_{conv} \tag{5}$$

This means h$_{conv}$ = 63.4W/m^2.

2.2 Part 2

Calculate how much heat is transferred to the house through the solar roof: The heat transfer through the roof is equivalent to $\frac{\Delta T A k_{panel}}{L_{panel}}$ = 120kW

2.3 Part 3

If the house uses an air conditioning unit that requires 0.08 W of power per each W of heat that extracts from the house, what fraction of the power generated by the solar cells will be needed to keep the house at a constant 20 °C?
120kW heat transfer means 0.08W/t * 120kW to maintain house temperature, so AC power consumption is 9.6kW. Power generation of the panels is 0.5 (absorptivity) * 1000W (G) * 0.2 (20 percent conversion) * 100m^2 (area) = 10000kW, and $\frac{9.6kW}{10kW}$ * 100 percent = 96 percent.

2.4 Part 4

Use Matlab to generate a plot of the temperature of the top panel surface versus the internal temperature. Assume that the heat transfer coefficient calculated in (1) remains constant. Use at least 100 temperature points between 15 to 25 °C. Please include axis labels and submit a script that generates your plot along with your solutions PDF file.
My Equation Setup [Ti is a linspace, which requires the guess to also be linspace]:

$$fsolve(@(T_s)(k\frac{T_i - T_s}{L}+G_{rad}-h_{rad}[T_s-T_{sur}]-h_{conv}(T_s-T_\infty), linspace(a, b, 100)) \tag{6}$$

3 Problems from the Textbook

3.1 Problem 1.52

1.52 If a dryer is designed to operate with an electric power consumption of P_{elec} = 500 W and to heat air from an ambient temperature of T_i = 20 °C to a discharge temperature of T_o = 45 °C, at what volumetric flow rate should the fan operate? Heat loss from the casing to the ambient air and the surroundings may be neglected. If the duct has a diameter of D = 70 mm, what is the discharge velocity V_o of the air? The density and specific heat of the air maybe approximated as ρ = 1.10 kg/m^3 and c_p = 1007 J/kgK, respectively.

$P_{elec} = mc_p * (T_{out} - T_{in})$ = 500W = m * 1007J/kgK * 25C, so m = 0.0199 kg/s. Volumetric Flow Rate = m/ρ = 0.0181m^3/s. Volumetric Flow Rate = A * V_o so $\frac{0.0181m^3}{\pi * .035m^2 s} = V_o = 4.703\frac{m}{s}$.

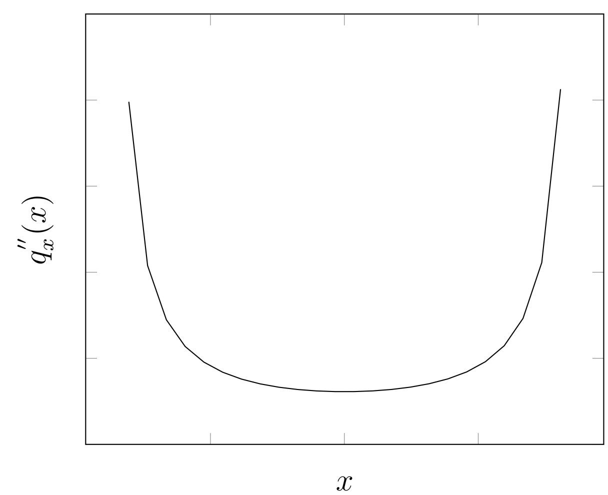
Consider a dryer duct length of L = 150 mm and a surface emissivity of ϵ = 0.8. If the coefficient associated with heat transfer by natural convection from the casing to the ambient air is h = 4 W/m^{2}K and the temperature of the air and the surroundings is $T_\infty = T_{sur}$ = 20 °C, confirm that the heat loss from the casing is, in fact, negligible. The casing may be assumed to have an average surface temperature of T_s = 40 °C.

Surface Area is roughly $2\pi(0.035m)L$. Using $q''_{conv} + q''_{rad} = h_{conv}A(T_s - T_\infty)$ $+ \epsilon\sigma A(T_s^4 - T_{sur}^4)$ = 2.643W, which is negligible compared to 500W power.

3.2 Problem 2.2

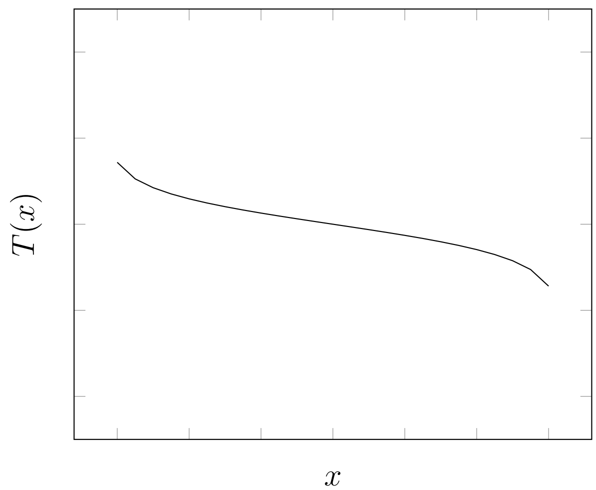
2.2 Assume steady-state, one-dimensional conduction in the axisymmetric object below, which is insulated around its perimeter. If the properties remain constant and no internal heat generation occurs, sketch the heat flux distribution, $q''_x(x)$, and the temperature distribution, T(x). Explain the shapes of your curves. How do your curves depend on the thermal conductivity of the material?

Cross-sectional area of object will increase in a way generalized by $b - ax^2$ (a and b > 0), where the center has the peak cross-sectional area while the start and end have a smaller one. Heat flux is inversely proportionate, so distribution will be the function $\frac{d}{b-ax^2}$.



Consequently, T(x) which is $\int \frac{q''_x(x)}{-k} dx$ will be the following equation:

$$T(x) = \frac{kd * ln\frac{|ax-\sqrt{ab}|}{|ax+\sqrt{ab}|}}{2\sqrt{a}\sqrt{b}} \tag{7}$$



A greater thermal conductivity k will make the temperature fall steeper, while a smaller k will make it more gradual.

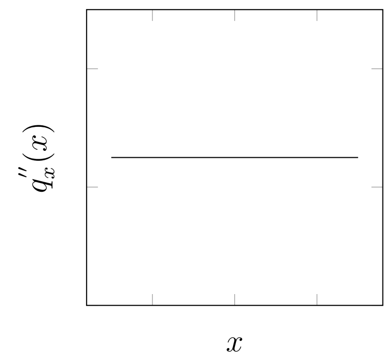
3.3 Problem 2.6

2.6 A composite rod consists of two different materials, A and B, each of length 0.5L. The thermal conductivity of Material A is half that of Material B, that is, $k_A/k_B = 0.5$. Sketch the steady-state temperature and heat flux distributions, T(x) and $q''_x(x)$, respectively. Assume constant properties and no internal heat generation in either material.

$$q''_x(x) = \frac{\Delta T}{L * R_{tot}} \tag{8}$$

$$R_{tot} = \frac{1}{k_A} + \frac{1}{k_B} \tag{9}$$

Since L, the start/end temperatures, k_A and k_B are constants, then $q_x''(x)$ is also a constant.



$T_1 < T_2$. Since cross-sectional area, k_A, k_B and $q_x''(x)$ are constants, and $2k_A = k_B$, then T(x) from 0 to L/2 will have half the slope of T(x) from L/2 to L.