Transport Homework 1

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1 Problem Statement

Humans are able to control their heat production rate and heat loss rate to maintain a nearly constant core temperature of $T_c = 310$ K under a wide range of environmental conditions. This process is called thermoregulation. From the perspective of calculating heat transfer between a human body and its surroundings, we focus on a layer of skin and fat, with its outer surface exposed to the environment and its inner surface at a temperature slightly less than the core temperature, $T_i = 308$ K. Consider a person with a skin/fat layer of thickness L = 3 mm and effective thermal conductivity k = 0.3 W/m K. The person has a surface area $A = 1.8m^2$ and is dressed in a bathing suit. The emissivity of the skin is $\epsilon = 0.95$.

Assumptions: The bathing suit has no effect. One dimensional heat transfer in fat/skin layer. Steady state conditions.

2 Problem Parts and Solutions

A. When the person is in still air in the middle of a summer day at T = 303 K, what is the skin surface temperature? Convection heat transfer to the air is characterized by a free convection coefficient of $h_{free} = 2W/m^2K$.

Energy Balance: $\dot{E}_{in} - \dot{E}_{out} = 0$. So for unit area, $q"_{conduction} - q"_{convection} - q"_{radiation} = 0$. Plugging in formulas for q, using Fourier's law for conduction, Newton's law of cooling for convection, and the radiation expression yields $k\frac{T_i - T_s}{L} = h_{free}(T_s - T_\infty) + \epsilon\sigma((T_s)^4 - (T_{sur})^4$. I then plugged this along with the given numbers into MATLAB's fsolve function (solving for Ts) to get 307.6K (script included in folder).

B. Given the hot summer temperatures, the person starts using a fan and notices that her skin surface temperature drops by 1K. What is the new convection coefficient, h_{fan}, imposed by the fan?

Going back to the formula from part A and using 307.6K for Ts. Also adding another convection term h_{fan} and solving for it. Via MATLAB's fsolve, this yields an h_{fan} value of about 31.9W/m^2K.

C. At what temperature would the environment need to be for the person to experience the same skin surface temperature as in question 2 without the fan (i.e., only free convection)?

This requires solving equation A with T_∞, which is also T_{sur} as the unknown, with Ts equal to 306.8K. Via MATLAB's fsolve, this yields a new T_{sur} of 288.7K.

*Initial calculations were done for per-unit area and the last result seemed weird to me, so I redid the calculations with Area $= 1.8m^2$ added onto the convection and radiation terms.